



Wesley College Mathematics Department

Semester One Examination 2010

3 C/D MAT

Name: _____

Teacher: _____

Section 1 : Resource Free (40 marks)

Time Allowed :

Reading time before commencing work: 5 minutes

Working time for this section: 50 minutes

Material required/recommended for this section:

To be provided by the supervisor:

This question booklet
Formula sheet

To be provided by the candidate:

Standard items: pens, pencils, eraser, correction fluid, ruler, highlighters

Mathematics 3CD MAT

Calculator Free

Structure of this section:

Question	Marks
1	7
2	4
3	4
4	5
5	8
6	9
7	3

Total Marks: **40**

Instructions to Candidates:

- Answer all questions
 - Spare pages are provided at the end of this booklet. If you need to use the spare paper, indicate clearly which question is being continued.
 - Show your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. **Correct answers given without supporting reasoning may not be allocated full marks.**
- Incorrect answers given without supporting reasoning cannot be allocated any marks.**
- It is recommended that you **do not use pencil** except in diagrams.

1. (1, 3, 3 = 7 marks)

Differentiate the following functions:

(i) $y = \frac{1}{4}x^4 - 9x + 2$

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(ii) $y = \sqrt{3x^2 - 8x}$

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(iii) $y = \frac{2e^{3x} - 4e^{-2x}}{e^x + e^x}$

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2. (4 marks)

If $f(x) = e^x \cdot x^2$ determine the exact value of $f'(1)$ showing all relevant working.

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3. (1, 3 = 4 marks)

Two events A and B are such that: $P(A) = 0.3$ $P(B) = x$ and $P(A \cup B) = 0.7$

For what value(s) of x are A, B:

(i) mutually exclusive

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(ii) independent

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4. (1, 2, 2, = 5 marks)

For the Friday afternoon golfing activity, students catch a bus from school to take them to Collier Park. It can be assumed that the length of time to between school and Collier Park follows a uniform distribution and takes between 8 and 12 minutes, depending on traffic.

(i) make a neat sketch of the probability density function

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(ii) determine the probability that last Friday, the bus took between 9 and 10 minutes given that it took less than 11 minutes

- (iii) write down a formulae for calculating $P(T > t)$ the probability that the time taken is more than t minutes.

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5. (1, 4, 3 = 8 marks)

(a) Functions f, h are defined: $f(x) = \frac{1}{x}$ $h(x) = \sqrt{2-x}$

- (i) determine $f \circ h(-7)$

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- (ii) define $h \circ f(x)$ and clearly state the (exact) domain and range of $h \circ f(x)$

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(b) Two functions f and g are such that: $g(x) = 1 - 2x$ and $g \circ f(x) = -4x^2 + 6x - 7$ Determine the function $f(x)$.

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6. (1, 1, 2, 2, 1, 2 = 9 marks)

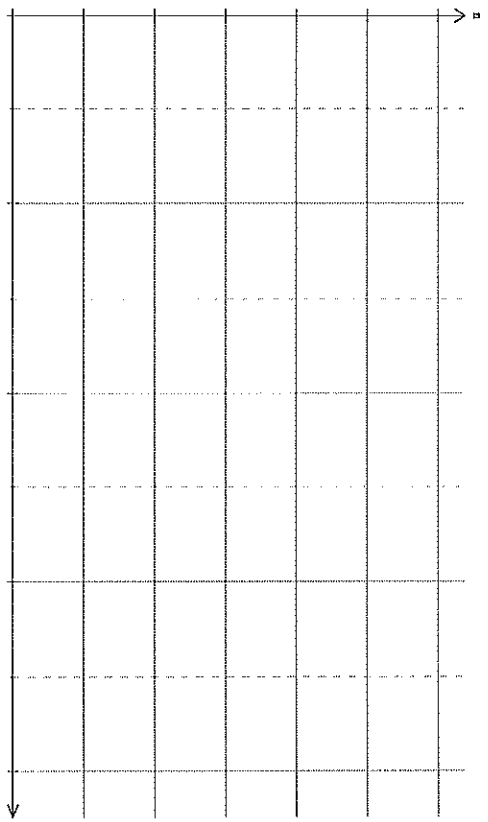
Foxes are baited to reduce their numbers in a nature reserve. Previous experiments indicate that the fox population (n), t weeks after baiting commenced, can be modelled by the function:

$$n = \frac{10000}{t+2} + 1000, \quad t \geq 0$$

(a) How many foxes were initially present in the reserve?

(b) How many foxes were present after 3 weeks?

(c) Sketch on the axes provided, the graph of n against t for $0 \leq t \leq 8$



(d) Determine an expression for the rate of change of the fox population

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- (e) At what rate is the fox population changing when $t = 3$

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- (f) What will happen to the fox population if baiting continued over a long time. Explain your answer.

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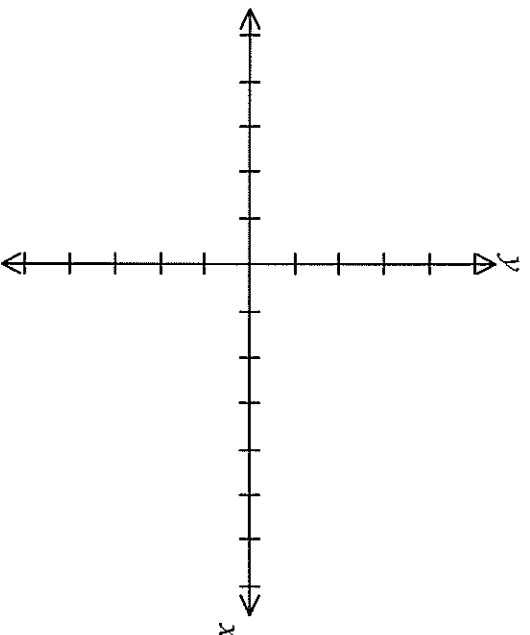
7. (3 marks)

A student attempting to sketch the graph of a function obtained the following information:

$$\frac{dy}{dx} = 0 \text{ for } x = -3, x = 2, x = 5 \quad \text{roots } 0, 4$$

$$\frac{d^2y}{dx^2} < 0 \text{ for } x = -3 \quad \frac{d^2y}{dx^2} > 0 \text{ for } x = 2 \text{ and } \frac{d^2y}{dx^2} = 0 \text{ for } x = 5$$

Use the student's information to make a sketch of a possible function.



END OF SECTION 1



Wesley College Mathematics Department

Semester One Examination 2010

3 C/D MAT

Name: _____

Teacher: _____

Section 2 : Calculator Assumed (80 marks)

Time Allowed :

Reading time before commencing work: 5 minutes

Working time for this section: 100 minutes

Material required/recommended for this section:

To be provided by the supervisor:

This question booklet

Formula sheet (retained from Section One)

To be provided by the candidate:

Standard items: pens, pencils, eraser, correction fluid, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, two graphics calculators/classpads that satisfy the conditions set by the Curriculum Council and a scientific calculator.

Mathematics 3CD MAT

Calculator Assumed

Structure of this section:

Question	Marks
1	4
2	4
3	10
4	9
5	4
6	5
7	8
8	8
9	7
10	9
11	5
12	7

Total Marks: 80

Instructions to Candidates:

- Answer all questions
- Spare pages are provided at the end of this booklet. If you need to use the spare paper, indicate clearly which question is being continued.
- Show your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. **Correct answers given without supporting reasoning may not be allocated full marks.**
- **Incorrect answers given without supporting reasoning cannot be allocated any marks.**
- It is recommended that you **do not use pencil** except in diagrams.

1. (4 marks)

Determine the coordinates of the point(s) on the curve $y = x^2 + \frac{16}{x}$ where the tangent is parallel to the x-axis.

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2. (4 marks)

Two quantities are related by the formulae : $Q = \frac{2}{\sqrt{x}}$ Use the incremental change formulae $\frac{\Delta Q}{\Delta x} \approx \frac{dQ}{dx}$ to estimate the percentage change in Q caused by a 10% reduction in x .

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3. (1, 2, 3, 2, 2 = 10 marks)

(a) In order for customers of a bank to access on-line internet banking, a 5-part entry code ***** consisting of letters of the alphabet or digits (0 → 9) or a combination of both letters and digits has to be used. Repetitions of letters and digits is allowed.

(i) how many such codes are possible?

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(ii) how many codes alternate between digits and letters?

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The bank now decides to change the rules by insisting that the code contain at least 1 letter and at least 1 digit.

(iii) how many codes are now possible?

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(b)

Another bank asked their staff to create 5-part entry codes ***** to allow access to secure parts of the bank. The staff could choose to use digits or letters or both, but **repetitions were not allowed**. If a staff member from this bank was randomly selected, determine the probability that their code consisted of:

(i) 3 letters and 2 digits

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(ii) at least 1 digit

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4. (3, 3, 3 = 9 marks)

A pathology service performs blood tests to detect the presence of a certain type of enzyme E. For 4% of blood samples with enzyme E, the tests suggest its absence (that is, it tests negative); while for 8% of samples without the enzyme, the test suggests its presence (that is, it tests positive). From past data, it is known that 25% of all samples received have the enzyme. Suppose that one of the fresh samples is taken at random and tested for enzyme E. Calculate the probability that the sample:

(i) tests positive

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(ii) contains enzyme E given that the test is positive

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(iii) tests positive or has the enzyme in it.

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5. (4 marks)

The function $p(x)$ is defined in terms of another function $h(x)$ such that:

$$p(x) = \frac{h(x)-1}{h(x)+1} \quad \text{In addition, it is known that } h(1) = 3, \quad h'(1) = 4$$

determine the value of $p'(1)$

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6. (5 marks)

A particle moves in rectilinear motion with equation $x = t^3 - 9t^2 + 24t - 60$ Showing relevant working, calculate the total distance covered between $t = 0$ and $t = 5$.

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7. (1, 2, 2, 3 = 8 marks)

When a biased six-faced die is rolled, the value, X , on the uppermost face follows this probability distribution:

x	1	2	3	4	5	6
$P(X = x)$	0.1	0.2	0.2	0.1	0.1	0.3

Calculate the probability that:

(i) in two rolls of the die, a “3” is followed by a “6”

(ii) in three rolls of the die, “1”, “2” and “3” occur in any order

(iii) at least one “6” occurs in 5 rolls of the die

(iv) in two rolls of the die the sum of the numbers obtained is six.

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8. (2, 1, 2, 1, 2 = 8 marks)

A machine is set to fill packets of potato chips with 200g of chips. However, due to inaccuracy of this type of machine, the actual weights in packets are normally distributed with a mean of 201g and a standard deviation of 4.5g. A quality control measure used by the factory is to weigh each packet after filling and recycle any packet less than 195g.

(a) What percentage of packets will be recycled?

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(b) If the factory produces 12 000 packets per day, how many will be recycled in one day?

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(c) If a packet is selected from those destined for recycling, what is the probability that its weight is less than 190g?

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(d) If 10 packets are randomly selected from those destined for recycling, use an appropriate probability distribution to determine the probability that:

(i) exactly 2 weigh less than 190 g

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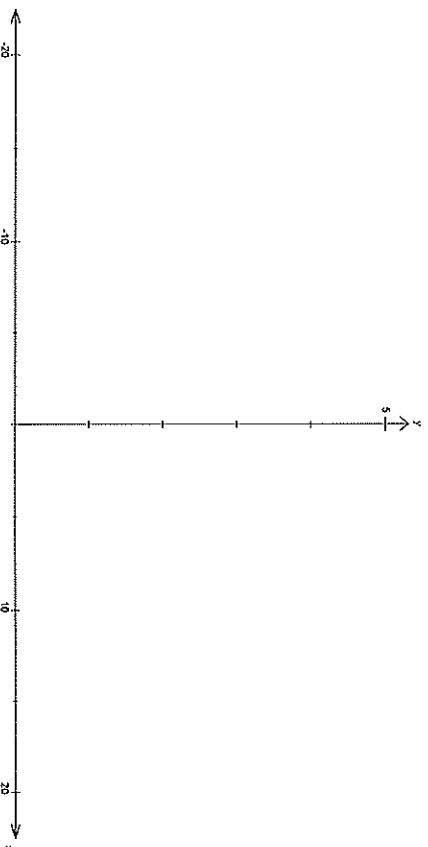
(ii) more than 2 weigh less than 190 g

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9. (2, 1, 2, 1, 1 = 7 marks)

The curve $y = e^{\frac{x}{20}} + e^{-\frac{x}{20}}$ is an example of a catenary curve, found in the shape of suspended power poles and cables, chains and in the (inverted) shape of the Gateway Arch in the city of St Louis.

(a) Sketch the graph $y = e^{\frac{x}{20}} + e^{-\frac{x}{20}}$ for $-20 \leq x \leq 20$ on the axes below



(b) Locate the minimum point, writing the coordinates on your graph above.

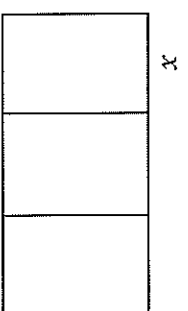
(c) Identify the endpoints ($x = \pm 20$) which correspond to the poles or pylons supporting the line.

(d) What is the maximum sag below the horizontal?

(e) Engineers from Perth wish to construct a similar structure and apply a horizontal dilation of factor 2 to the equation of the catenary curve given. Write down the equation of the curve to be used in Perth.

10. (1, 1, 1, 2, 4 = 9 marks)

A business centre is to have a total floor space of $600m^2$ and is to be divided into three rectangular rooms of equal size (each of length x and width y as shown in the diagram).



All of the walls will cost \$60 per metre to build.

- (i) write an expression for the total area in terms of x and y

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- (ii) write x in terms of y

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- (iii) write an expression for the perimeter in terms of y

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- (iv) show that the cost \$C is given by $C = \frac{72000}{y} + 240y$

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- (v) use calculus methods to determine the dimensions of the business centre that minimize the cost of the walls.

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11. (5 marks)

A police radar unit is used to measure the speed of vehicles travelling along Coode Street which has a speed limit of 60 km/hr. The recorded speeds of vehicles are normally distributed. Determine the mean and standard deviation of speeds if it is known that 4% of all vehicles travel faster than 70 km/hr and only 16% of all vehicles travel slower than the speed limit.

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12. (1, 1, 2, 3 = 7 marks)

The rate at which oil is leaking from a damaged drum is found to be a constant number k multiplied by the amount of oil which is left in the drum. The drum is full and contains A_0 litres before the leak starts and has A litres remaining after t hours.

(a) Write a differential equation to represent this situation

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(b) Write an exponential equation to represent this situation

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(c) If 10% leaks out in the first hour, find the value of k to 4 decimal places

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(d) If there is only 15 litres left after 18 hours, determine the capacity of the drum to the nearest litre.

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END OF EXAMINATION